AQ8.1: Activity Questions 1 - Not Graded

**This assignment will not be graded and is only for practice.**

 Continuity at a point:**Continuity at a point:**  
A function f*f* is said to be continuous at a point x=a*x*=*a* in its domain, if  
1) f(a) exists; i.e value of f(x)*f*(*x*) at x=a*x*=*a* exists.  
2) lim⁡x→af(x)*x*→*a*lim​*f*(*x*) exists; i.e both left and right limits exist and are equal.  
3) lim⁡x→af(x)=f(a)*x*→*a*lim​*f*(*x*)=*f*(*a*).  
  
Equivalently,  
  
we can say that f*f* is continuous**continuous** at x=a*x*=*a*, if {xn}→a{*xn*​}→*a* implies that {f(xn)}→f(a){*f*(*xn*​)}→*f*(*a*).

* A function f:X→Y*f*:*X*→*Y* is bounded if we can find two real numbers m*m* and M*M* such that m<f(x)<M*m*<*f*(*x*)<*M* for all x∈X*x*∈*X*, where X*X* and Y*Y* are two subsets of RR.
* The greatest integer function ⌊⌋:R→Z⌊⌋:R→Z of a real number x*x* denotes the greatest integer less than or equal to x*x*.
* The smallest integer function ⌈⌉:R→Z⌈⌉:R→Z of a real number x*x* denotes the least integer greater than or equal to x*x*.

**Level 1:**

***1 point***

If lim⁡x→af(x)=L*x*→*a*lim​*f*(*x*)=*L*, which of the following statements must be true?

f*f* is defined at a*a*.

lim⁡x→a−f(x)=L*x*→*a*−lim​*f*(*x*)=*L*

f(a)=L*f*(*a*)=*L*

lim⁡x→a−f(x)=L*x*→*a*−lim​*f*(*x*)=*L*

f*f* is continuous at a*a*

***1 point***

Let f*f* be a function and the Figure M2W2AQ1 represent the graph of function f*f*. The solid points denote the value of the function at the points, and the values denoted by the hollow points are not taken by the functions.  
  
A graph of a function

AI-generated content may be incorrect.  
  
Choose the set of correct options.

lim⁡x→2−f(x)=10*x*→2−lim​*f*(*x*)=10

lim⁡x→2+f(x)=2*x*→2+lim​*f*(*x*)=2

lim⁡x→4−f(x)=4*x*→4−lim​*f*(*x*)=4

lim⁡x→4+f(x)=4*x*→4+lim​*f*(*x*)=4

f*f* is continuous at x=2*x*=2.

f*f* is continuous at x=6*x*=6.

Consider the following function:  
  
  
f(x)={x(x+1)(x+2)if x≥116if x<1,*f*(*x*)={(*x*+1)(*x*+2)*x*​61​​if *x*≥1if *x*<1,​  
The number of discontinuity of f*f* is

***1 point***

***1 point***

Define a function f*f* as follows:  
  
  
f(x)={axif x≥1bx2if x<1,*f*(*x*)={*axbx*2​if *x*≥1if *x*<1,​  
  
  
where a*a* and b*b* are two arbitrary real numbers.

f*f* is always continuous.

f*f* is continuous if and only if a=b=0*a*=*b*=0.

f*f* is continuous if a=b=0*a*=*b*=0.

f*f* is continuous if a=b*a*=*b*.

 **Level 2:**

***1 point***

Which of the following option(s) is(are) true?

There exists a continuous function f*f* from [1,5][1,5] to [1,7][1,7] such that f(3)=π*f*(3)=*π*.

There exists a continuous function f*f* from [1,10][1,10] to [2,8][2,8] such that f(1)=3*f*(1)=3 and f(10)=5*f*(10)=5.

There exists a non-constant continuous function f*f* from RR to RR which is bounded.

There exists an unbounded continuous function from (2,3](2,3] to RR.

***1 point***

Consider a function f:R→R*f*:R→R such that f(cx)=cf(x)*f*(*cx*)=*cf*(*x*) for all c,x∈R*c*,*x*∈R. Which of the following option(s) is(are) correct?

f(x+y)=f(x)+f(y)*f*(*x*+*y*)=*f*(*x*)+*f*(*y*) for all x,y∈R*x*,*y*∈R

f*f* is not continuous in RR.

f*f* is continuous in RR.

lim⁡x→af(x)*x*→*a*lim​*f*(*x*) exists for all a∈R*a*∈*R*, but f*f* is not continuous in RR.

Define a function f*f* as follows:  
  
  
f(x)={xtan−12xif x>0bif x=0sin(ax)xif x<0,*f*(*x*)=⎩⎨⎧​*tan*−12*xx*​*bxsin*(*ax*)​​if *x*>0if *x*=0if *x*<0,​  
  
  
If f*f* is continuous then the value of a+b*a*+*b* is

***1 point***

***1 point***

Define a function f*f* as follows:  
  
  
f(x)={1e1x+1if x≠0,0if x=0*f*(*x*)=⎩⎨⎧​*ex*1​+11​0​if *x*=0,if *x*=0​  
  
  
Which of the following option(s) is(are) true?

lim⁡x→0−f(x)≠lim⁡x→0+f(x)*x*→0−lim​*f*(*x*)=*x*→0+lim​*f*(*x*)

lim⁡x→0−f(x)=lim⁡x→0+f(x)*x*→0−lim​*f*(*x*)=*x*→0+lim​*f*(*x*)

f*f* is a bounded function on RR.

f*f* is continuous at x=0*x*=0.